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Global Damping of a Vibration Structure with a Locally Controlled Absorber

Jing Yuan*
Hong Kong Polytechnic University, Hong Kong, People's Republic of China

The dynamic vibration absorber is a well-known vibration control device. Its performance can be enhanced by a proper synthesis of the coupling force between the absorber mass and the primary structure. This study proposes a locally controlled absorber that uses only a feedback signal measured at the coupling point. The absorber is based on an identified transfer function and tolerates identification errors. It is able to introduce global damping into the closed-loop primary structure, and it absorbs vibration at tuned frequencies. Simulation results are presented to verify the theoretical results.

Introduction

THE dynamic vibration absorber is a well-known vibration-control device.[1] It can be improved by adding an active coupling force between the absorber mass and the primary structure. The absorber controller may use global-state feedback[2–5] or a local feedback signal[6–9] to generate its control force. The second kind of absorbers are known as "locally controlled absorbers" (LCAs); they are simpler and more economical. Most LCA schemes are analyzed by assuming single-degree-of-freedom (DOF) primary systems. When the primary structure is a multiple spring–mass chain with a tridiagonal stiffness matrix, a LCA may extend its absorption effect from the coupling point to other points,[9] provided that the model parameters are known exactly.

In practice, the stiffness matrix of a model is not necessarily tridiagonal[10] and mismatch exists between models and primary structures. Effects of available LCAs are limited to the coupling points under such conditions. In many cases, a LCA may absorb vibration at a frequency at the expense of adding new resonant peaks to the primary structure. A new LCA is proposed here to solve the existing problems.

The proposed LCA is based on an identified model and tolerates identification errors. It is able to introduce global damping in to the primary structure and absorb vibration at tunable frequencies without being restricted to multiple spring–mass chains with tridiagonal stiffness matrices. In a simulation example, the proposed LCA is applied to a flexible plate to absorb vibration and damp the resonant peaks simultaneously.

Mathematical Model

The dynamics of a multiple spring–mass system may be expressed in the Laplace transform domain as

$$\boldsymbol{A}(s)\boldsymbol{x} = [\boldsymbol{M}s^2 + \boldsymbol{C}s + \boldsymbol{K}]\boldsymbol{x} = \boldsymbol{d} \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are $m \times m$ mass, damping, and stiffness matrices, $\boldsymbol{x}$ is an m-DOF vector, and

$$\boldsymbol{d}^T = [\underbrace{0 \quad \cdots \quad 0}_{j-1\ \text{Zeros}}\, d_j\, \underbrace{0 \quad \cdots \quad 0}_{m-j\ \text{Zeros}}]$$

is the disturbance vector acting on the jth DOF of the structure. In the case of multiple disturbance sources, one may add more vectors like $\boldsymbol{d}$ to the right-hand side of Eq. (1) and analyze the responses by the same method presented here.

Alternatively, Eq. (1) could be derived by the modal theory. In any case, the model is for analysis only, and its parameters need not be known for the design of the LCA. The absorber is a mass m_a attached to the ith DOF of the primary system as shown in Fig. 1. The active coupling force (not plotted in the figure) is given by

$$f_i = -G(s)x_i + k_a(x_a - x_i) \tag{2}$$

Here $G(s)$ is a transfer function to be designed in this paper. x_i and x_a are displacements of the ith DOF and the absorber, respectively. The stiffness of the coupling spring is denoted by k_a. The LCA changes the dynamics of Eq. (1) to

$$\boldsymbol{A}(s)\,\boldsymbol{x} = \boldsymbol{f} + \boldsymbol{d} \tag{3}$$

where

$$\boldsymbol{f}^T = [\underbrace{0 \quad \cdots \quad 0}_{i-1\ \text{Zeros}}\, f_i\, \underbrace{0 \quad \cdots \quad 0}_{m-i\ \text{Zeros}}]$$

This equation has a local solution:

$$x_i = \frac{\det[\boldsymbol{A}_{i,i}(s)]f_i + \det[\boldsymbol{A}_{j,i}(s)]d_j}{\det[\boldsymbol{A}(s)]} \tag{4}$$

where $\boldsymbol{A}_{j,i}(s)$ is an $(m-1)\times(m-1)$ matrix obtained by removing the jth row and the ith column from $\boldsymbol{A}(s)$. Combining Eq. (2) with $x_a = -f_i/m_a s^2$, one obtains

$$f_i = -s^2\frac{k_a + G(s)}{s^2 + \omega_a^2}x_i \tag{5}$$

where $\omega_a^2 = k_a/m_a$ is the absorber resonance frequency. Substituting Eq. (5) into Eq. (4), one can express x_i as

$$x_i = \frac{\left(s^2+\omega_a^2\right)\det[\boldsymbol{A}_{j,i}(s)]d_j}{\left(s^2+\omega_a^2\right)\det[\boldsymbol{A}(s)] + s^2\det[\boldsymbol{A}_{i,i}(s)][k_a + G(s)]}$$
$$= \frac{\left(s^2+\omega_a^2\right)}{D(s) + s^2N(s)G(s)}\det[\boldsymbol{A}_{j,i}(s)]d_j \tag{6}$$

where $N(s) = \det[\boldsymbol{A}_{i,i}(s)]$ and

$$D(s) = \left(s^2+\omega_a^2\right)\det[\boldsymbol{A}(s)] + s^2k_a\det[\boldsymbol{A}_{i,i}(s)]. \tag{7}$$

A clear indication of Eq. (6) is $x_i = 0$ when $s = j\omega_a$, which is a common feature of all dynamic vibration absorbers. The absorption frequency ω_a is tunable by adjusting m_a and k_a. When the active part of Eq. (2) is turned off with $G(s) = 0$, the denominator of Eq. (6)

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*Associate Professor, Department of Mechanical Engineering, Hung Hom, Kowloon; mmjyuan@polyu.edu.hk.

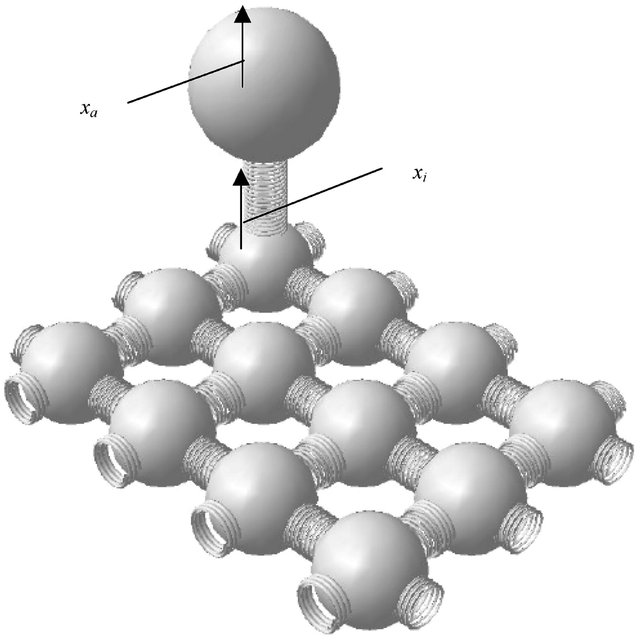

Fig. 1 Absorber attached to the *i*th DOF of a multiple spring–mass system.

becomes Eq. (7) with $m+1$ lightly damped roots. Passive absorption comes at the expense of an extra resonant peak in the primary system.

There are many methods for LCA design in the literature. Each one would lead to a possible version of $G(s)$ with different effects on the denominator of Eq. (6). The proposed LCA is designed with a focus on the denominator of Eq. (6), which is also the closed-loop characteristic equation of the vibration system. The design objective is to damp the vibration system by placing closed-loop eigenvalues in a desired region.

Proposed LCA

The active part of LCA depends on $\det[\boldsymbol{A}_{i,i}(s)]$, $\det[\boldsymbol{A}(s)]$, and a single feedback x_i. In the absence of a disturbance $\boldsymbol{d}$, Eq. (4) reduces to

$$x_i = \frac{\det[\boldsymbol{A}_{i,i}(s)]}{\det[\boldsymbol{A}(s)]} f_i \tag{8}$$

One may attach the absorber to a primary system and identify a transfer-function model for the design of the proposed LCA. In that case, Eq. (2) becomes

$$f_i = -u + k_a(x_a - x_i) \tag{9}$$

where u is a pseudo-random probing signal. A substitution of $x_a = -f_i/m_a s^2$ into Eq. (9) yields

$$f_i = -s^2 \frac{u + k_a x_i}{s^2 + \omega_a^2} \tag{10}$$

The combination of Eqs. (8) and (10) leads to

$$x_i = \frac{s^2 \det[\boldsymbol{A}_{i,i}(s)]}{\left(s^2 + \omega_a^2\right)\det[\boldsymbol{A}(s)] + s^2 k_a \det[\boldsymbol{A}_{i,i}(s)]} u = s^2 \frac{N(s)}{D(s)} u \tag{11}$$

where $N(s)$ and $D(s)$ are identifiable by any algorithm available in the literature.[11] The active part of the proposed LCA depends on a transfer function:

$$G(s) = R(s)/Q(s) \tag{12}$$

where $R(s)$ and $Q(s)$ are solvable from Eq. (14). The control effect becomes clear if one substitutes Eq. (12) into Eq. (6). The result reads

$$x_i = \frac{\left(s^2 + \omega_a^2\right)Q(s)}{D(s)Q(s) + s^2 N(s)R(s)} \det[\boldsymbol{A}_{j,i}(s)] d_j \tag{13}$$

One may place the closed-loop poles via a prototype polynomial $P(s)$ such that

$$D(s)Q(s) + s^2 N(s)R(s) = P(s) \tag{14}$$

The equation is solvable if $D(s)$ and $N(s)$ are coprime. If not, one can write $N(s) = N_1(s)C(s)$ and $D(s) = D_1(s)C(s)$, where $C(s)$ is a common divider. Only $N_1(s)$ and $D_1(s)$ are needed for the design of $G(s)$. The denominator of Eq. (6) will become $C(s)[D_1(s) + s^2 N_1(s)G(s)] = C(s)P(s)$, if one substitutes $N_1(s)$ and $D_1(s)$ for $N(s)$ and $D(s)$ in Eq. (14) to solve $R(s)$ and $Q(s)$. The LCA introduces damping to those resonant peaks that are related to the roots of $D_1(s)$. Roots of $C(s)$ are related to resonant peaks uncontrollable by the LCA due to the mounting location.

In reality, the LCA is attached to a point where vibration is most significant. Roots of $D_1(s)$ are related to significant resonant peaks. Active damping of these peaks is an important objective. In an extreme case, the LCA can be attached to the free end of a one-dimensional cantilever beam; then Eq. (11) represents a controllable plant, and $N(s)$ and $D(s)$ will be coprime. The LCA matches the closed-loop characteristic equation to prototype polynomial $P(s)$. Selection of $P(s)$ will be explained in the next section, which links the denominator of Eq. (6) to the closed-loop characteristic equation to demonstrate the global damping effect.

Closed-Loop Eigenvalues

By focusing on the coupling between the ith DOF and the absorber, one may describe the LCA effect as

$$\begin{bmatrix} a_{i,i}(s) + k_a + G(s) & -k_a \\ -k_a - G(s) & m_a s^2 + k_a \end{bmatrix} \begin{bmatrix} x_i \\ x_a \end{bmatrix} \tag{15}$$

where $a_{i,i}(s)$ is the ith diagonal entry of $\boldsymbol{A}(s)$. A permutation is applied to Eq. (1) to switch the ith row and column of $\boldsymbol{A}(s)$ with their respective mth counterparts. The ith DOF of $\boldsymbol{x}$ and $\boldsymbol{d}$ are also interchanged with their respective mth counterparts. The dynamic equation remains valid, and the $\det[\boldsymbol{A}(s)]$ is not affected by the permutation. The closed-loop system now includes the primary structure and the absorber, which may be expressed as

$$\boldsymbol{A}^*(s)\boldsymbol{x}^* = \boldsymbol{d}^* \tag{16}$$

The absorber displacement x_a and a zero entry have been appended to $\boldsymbol{x}$ and $\boldsymbol{d}$, respectively, to form the extended vectors $\boldsymbol{x}^*$ and $\boldsymbol{d}^*$. The extended system matrix $\boldsymbol{A}^*(s)$ has the form

$$\boldsymbol{A}^*(s) = \begin{bmatrix} a_{1,1}(s) & \cdots & a_{1,m-1}(s) & a_{1,m}(s) & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{m-1,1}(s) & \cdots & a_{m-1,m-1}(s) & a_{m-1,m}(s) & 0 \\ a_{m,1}(s) & \cdots & a_{m,m-1}(s) & a_{m,m}(s) + k_a + G(s) & -k_a \\ 0 & \cdots & 0 & -k_a - G(s) & m_a s^2 + k_a \end{bmatrix} \tag{16a}$$

where $a_{i,j}(s)$ denotes the (i, j)th entry of matrix $\boldsymbol{A}(s)$ after permutation. To avoid excessive symbols, no new symbols are used to denote the permuted system matrix and its entries.

The intersection of the last two rows and last two columns of $\boldsymbol{A}^*(s)$ is equivalent to the coupling matrix of Eq. (15) after the permutation. Expressing $\det[\boldsymbol{A}^*(s)]$ with respect to the last row of $\boldsymbol{A}^*(s)$, one can write

$$\det[\boldsymbol{A}^*(s)] = \left[m_a s^2 + k_a\right] \det\left[\boldsymbol{A}^*_{m+1,m+1}(s)\right] + [k_a + G(s)] \det\left[\boldsymbol{A}^*_{m+1,m}(s)\right] \tag{17}$$

where

$$A^*_{m+1,m+1}(s) = \begin{bmatrix} a_{1,1}(s) & \cdots & a_{1,m-1}(s) & a_{1,m}(s) \\ \vdots & \ddots & \vdots & \vdots \\ a_{m-1,1}(s) & \cdots & a_{m-1,m-1}(s) & a_{m-1,m}(s) \\ a_{m,1}(s) & \cdots & a_{m,m-1}(s) & a_{m,m}(s) + k_a + G(s) \end{bmatrix} \tag{17a}$$

$$A^*_{m+1,m}(s) = \begin{bmatrix} a_{1,1}(s) & \cdots & a_{1,m-1}(s) & 0 \\ \vdots & \ddots & \vdots & \vdots \\ a_{m-1,1}(s) & \cdots & a_{m-1,m-1}(s) & 0 \\ a_{m,1}(s) & \cdots & a_{m,m-1}(s) & -k_a \end{bmatrix} \tag{17b}$$

Equation (17b) indicates $\det[A^*_{m+1,m}(s)] = -k_a \det[A_{m,m}(s)]$. Substituting this into Eq. (17), one obtains

$$\det[A^*(s)] = \left[m_a s^2 + k_a\right] \det\left[A^*_{m+1,m+1}(s)\right] - k_a[k_a + G(s)] \det[A_{m,m}(s)] \tag{18}$$

On the other hand, Eq. (17a) suggests

$$\det\left[A^*_{m+1,m+1}(s)\right] = (a_{m,m} + k_a + G)\det[A_{m,m}] + \sum_{k=1}^{m-1}(-1)^{m+k} a_{m,k} \det[A_{m,k}] = (k_a + G)\det[A_{m,m}] + \det[A(s)]$$

When substituted into Eq. (18), this means

$$\det[A^*(s)] = \left[m_a s^2 + k_a\right]\det[A(s)] + m_a s^2[k_a + G(s)]\det[A_{m,m}(s)] \tag{19}$$

Equation (19) differs from the denominator of Eq. (6) only by a multiplier m_a and labeling "m" vs "i." Because the permutation changes $\det[A_{i,i}(s)]$ to $\det[A_{m,m}(s)]$ without any effects on $\det[A(s)]$, Eq. (19) turns out to be the denominator of Eq. (6). This fact enables the proposed LCA to modify the closed-loop eigenvalues of the flexible structure via a prototype polynomial $P(s)$. It is analytically equivalent to damping vibration by specifying the closed-loop eigenvalues. This is a new feature not seen with other LCA schemes.

Simulation Results

A simulation was conducted to verify the effects of the proposed LCA. The flexible structure is a rectangular plate with length $L = 2$, width $W = 1$, and rigidly clamped edges. The plate has eigenfunctions $\phi_{nm}(x, y) = \sin(nx\pi/W)\sin(my\pi/L)$ and resonant frequencies:

$$\omega_{nm} = \pi\sqrt{(D/\rho h)[m^2/L^2 + n^2/W^2]}$$

with $D/\rho h = 0.25$ to scale the numerical range of simulation. The LCA, with $m_a = 1$ and $k_a = \omega_{11}^2$, is attached to $\boldsymbol{x}_i = [1, 0.5]^T$ to absorb vibration at $\omega_a = \omega_{11}$. The active part of the LCA is able to damp resonance for odd n and m, due to its mounting location. The model is truncated to $n, m \le 5$, which is practically possible by using antialias filters.

In the simulation, the active part of the LCA was turned off to simulate a passive vibration first. Figure 2 plots the normalized power spectral densities (PSD) of the unabsorbed (dotted) and absorbed (solid) vibration signals. The passive vibration absorption at ω_{11} comes with the price of an extra low-frequency resonant peak in the vibration system. Other resonant peaks are pushed to higher frequencies.

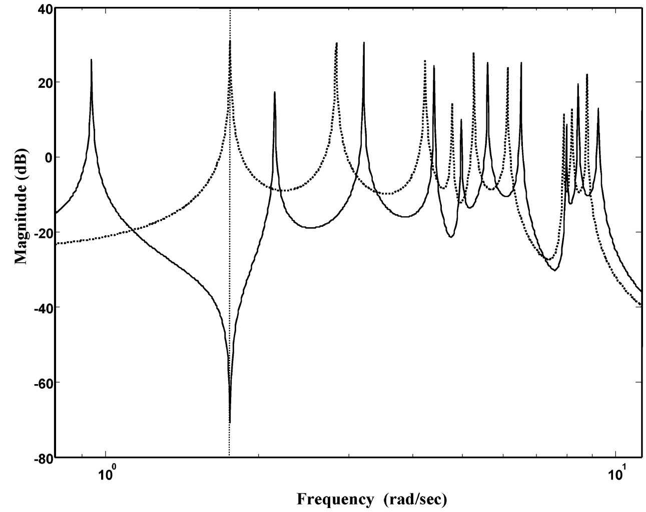

Fig. 2 Passive vibration absorption (——) vs unabsorbed vibration (. . . .).

The proposed LCA is intended to preserve the attenuation at $\omega_a = \omega_{11}$ while reducing the resonant peaks. It depends on a discrete-time model of the primary system identified by procedures similar to the descriptions of Eqs. (9–11). The result is a discrete-time version of Eq. (11) given by

$$x_i = [\hat{N}(z)/\hat{D}(z)]u_i$$

with identification errors $\Delta N = N(z) - \hat{N}(z)$ and $\Delta D = D(z) - \hat{D}(z)$. When combined with the disturbance, the complete model is

$$x_i = \frac{N(z)u_i + \left[z^2 - 2\cos(\omega_a\delta)z + 1\right]M(z)d_j}{D(z)} \tag{20}$$

where δ is the sampling interval and $[z^2 - 2\cos(\omega_a\delta)z + 1]M(z)d_j$ describes the effects of disturbance and the passive part of the LCA. The LCA transfer function $G(z)$ is given by a discrete-time version of Eq. (12) as

$$G(z) = R(z)/Q(z) \qquad \text{or} \qquad u_i = [-R(z)/Q(z)]x_i \tag{21}$$

where $R(z)$ and $Q(z)$ are solved from

$$\hat{D}(z)Q(z) + \hat{N}(z)R(z) = P(z) \tag{22}$$

and $P(z)$ is a polynomial with roots inside a disc with radius $\gamma < 1$. There are an infinite number of polynomials satisfying such a condition. Each of them is a candidate of $P(z)$ corresponding to a stable $G(z)$ via Eq. (22). When the second part of Eq. (21) is substituted into Eq. (20), the closed-loop dynamics become

$$x_i = \frac{Q(z)\left[z^2 - 2\cos(\omega_a\delta)z + 1\right]M(z)}{D(z)Q(z) + N(z)R(z)}d_j \tag{23}$$

The denominator of Eq. (23) is the closed-loop characteristic equation, which becomes

$$\hat{D}(z)Q(z) + \hat{N}(z)R(z) + \Delta DQ(z) + \Delta NR(z) = P(z) + \Delta DQ(z) + \Delta NR(z) = P(z)\left[1 + \frac{\Delta DQ(z) + \Delta NR(z)}{P(z)}\right] \tag{24}$$

upon substitution of Eq. (22). Identification errors ΔN and ΔD can be bounded by $\|\Delta N\|_\infty \le \delta_n$ and $\|\Delta D\|_\infty \le \delta_d$ by available system identification techniques.[11]

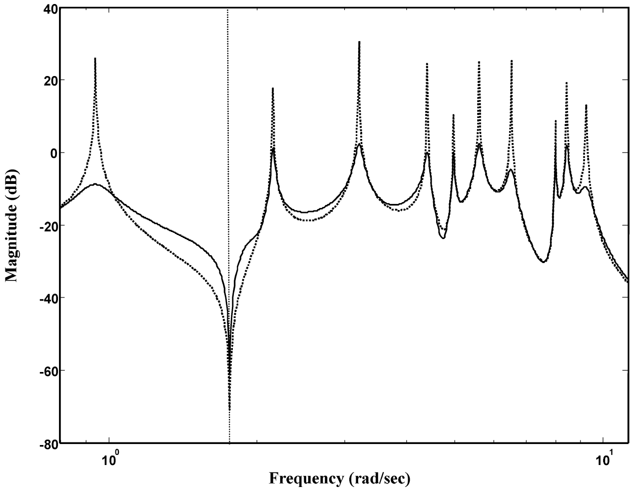

Fig. 3 Active (——) vs passive (. . . .) absorption.

In view of Eq. (24), $P(z)$ should be selected such that all roots of $P(z)$ are inside a disk with radius $\gamma < 1$ and

$$\left\| \frac{Q(z)\delta_d + R(z)\delta_n}{P(z)} \right\|_\infty \leq \gamma$$

Such an LCA will place and keep closed-loop eigenvalues within the disk with radius $\gamma < 1$ as long as $\|\Delta N\|_\infty \leq \delta_n$ and $\|\Delta D\|_\infty \leq \delta_d$. This is numerically solvable using the Matlab linear matrix inequality tool box.

Global damping of the proposed LCA is analytically possible by Eq. (22), which is a discrete-time version of Eq. (14) placing closed-loop eigenvalues in a stable region. The simulation signal for active absorption by the proposed LCA is plotted in Fig. 3 in comparison with its passive counterpart. The LCA preserves the attenuation valley while reducing the resonant peaks significantly, with only local feedback.

Conclusions

This study proposes an LCA with a global damping effect on a flexible structure. The new LCA is based on an identified transfer function and tolerates identification errors. It preserves the local absorption of its passive part while introducing active damping to the primary structure simultaneously. If necessary, it is able to absorb vibration at more tunable frequencies.

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References

[1]Hunt, J. B., *Dynamic Vibration Absorbers*, Mechanical Engineering Publishers, Ltd., London, 1979.

[2]Nonami, A. M., "Disturbance Cancellation Control for Vibration of Multi-Degree-of-Freedom Systems," *JSME International Journal*, Vol. 37, No. 1, 1994, pp. 86–93.

[3]Lee-Glauser, G. J., Juang, J.-N., and Sulla, J. L., "Optimal Active Vibration Absorber: Design and Experimental Results," *Journal of Vibration and Acoustics*, Vol. 117, No. 2, 1995, pp. 165–171.

[4]Lee-Glauser, G. J., Juang, J.-N., and Sulla, J. L., "Integrated Passive–Active Vibration Absorber for Multistory Buildings," *Journal of Structural Engineering*, Vol. 123, No. 3, 1997, pp. 499–504.

[5]Nagem, R. J., Madanshetty, S. I., and Imedhi, G., "An Electromechanical Vibration Absorber," *Journal of Sound and Vibration*, Vol. 200, No. 4, 1997, pp. 551–556.

[6]Olgac, N., and Holm-Hansen, B., "Tunable Active Vibration Absorber: The Delayed Resonator," *Journal of Dynamic Systems, Measurement and Control*, Vol. 117, No. 4, 1995, pp. 513–519.

[7]Burdisso, A., and, Heilmann, J. D., "A New Dual-Reaction Mass Dynamic Vibration Absorber Actuator for Active Vibration Control," *Journal of Sound and Vibration*, Vol. 214, No. 5, 1998, pp. 817–831.

[8]Filipovic, D., and Schroder, D., "Control of Vibrations in Multi-Mass Systems with Locally Controlled Absorbers," *Automatica*, Vol. 37, No. 2, 2001, pp. 213–220.

[9]Yuan, J., "Hybrid Dynamic Vibration Absorption by Zero/Pole Placement," *Journal of Vibration and Acoustics*, Vol. 122, No. 4, 2000, pp. 466–469.

[10]Seto, K., Ren, M., and Doi, F., "Modeling and Feedback Structural Acoustic Control of a Flexible Plate," *Journal of Vibration and Acoustics*, Vol. 123, No. 1, 2001, pp. 18–23.

[11]Goodwin, G. C., and Sin, K. S., *Adaptive Filtering, Prediction and Control*, Prentice–Hall, Englewood Cliffs, NJ, 1984, Chap. 7.

A. Berman
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